Bayesian Estimation of Trends in Population-Level Health Metrics Using Disparate Data Sources

Mariel McKenzie Finucane, PhD

MATHEMATICA Policy Research

WSS Seminar

November 12, 2014

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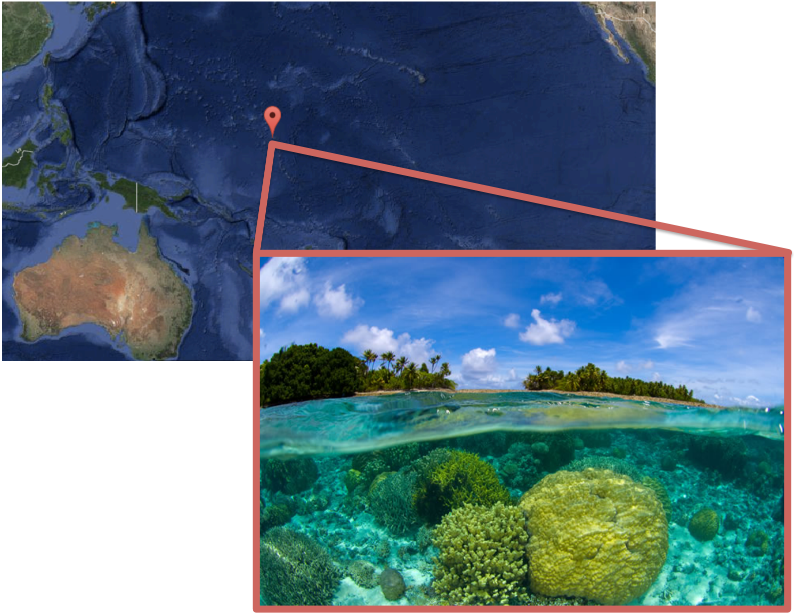
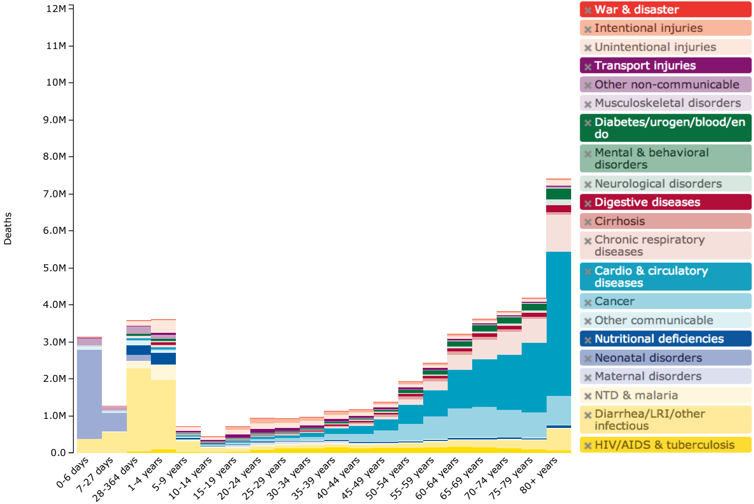


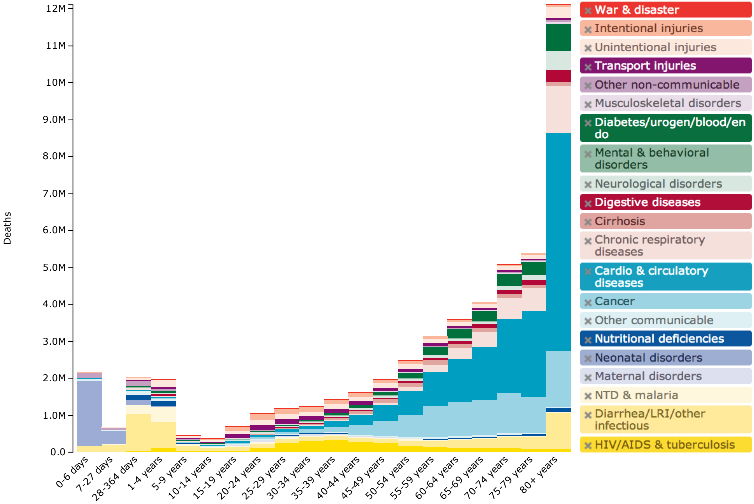
http://www.google.com/maps & http://traveltoanewcountry.com/oceania/marshall-islands.php

Deaths in 1990



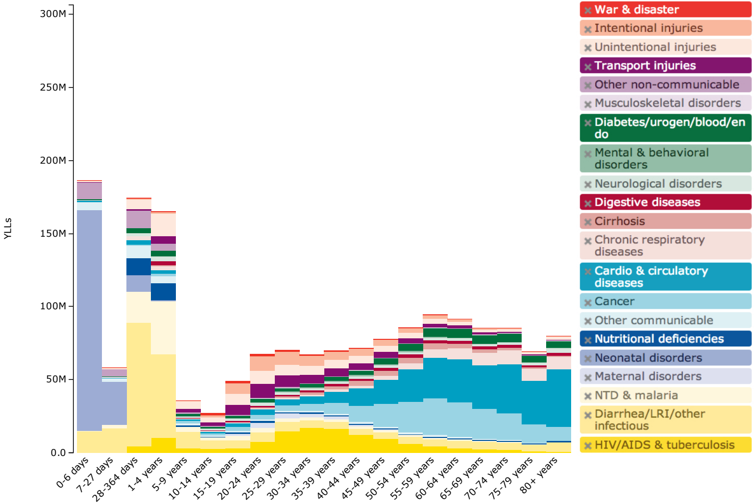
Lozano, Naghavi, ..., and Murray; *The Lancet*, 2012

Deaths in 2010



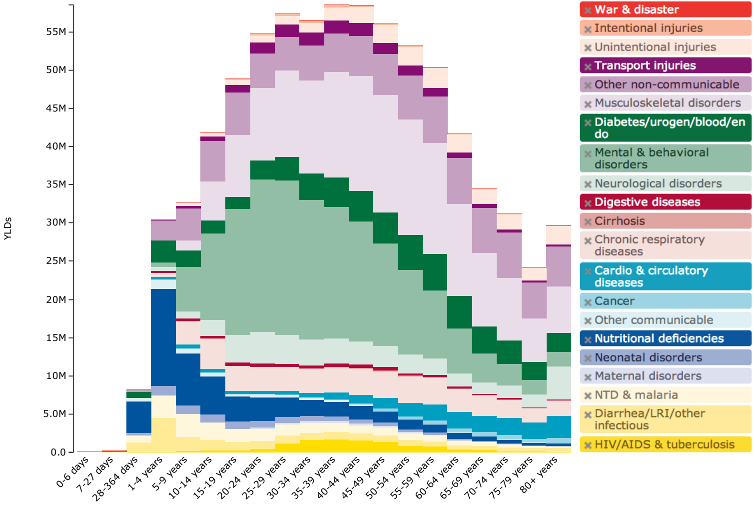
Lozano, Naghavi, ..., and Murray; *The Lancet*, 2012

Years of Life Lost in 2010



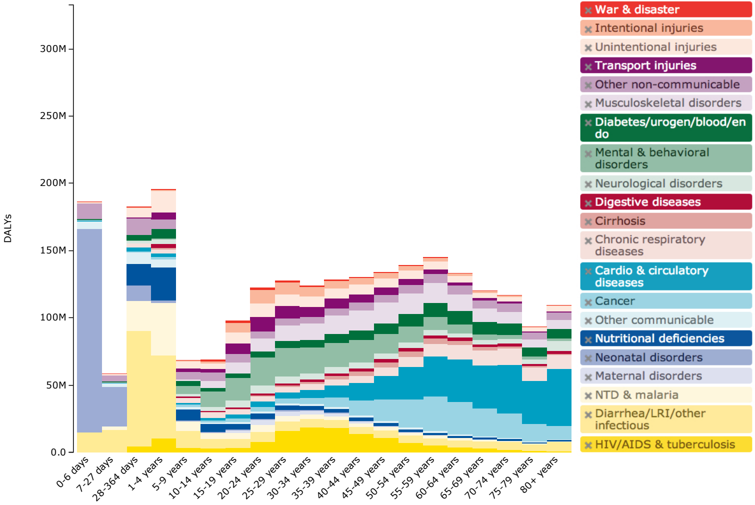
Lozano, Naghavi, ..., and Murray; *The Lancet*, 2012

Years Lost to Disability in 2010



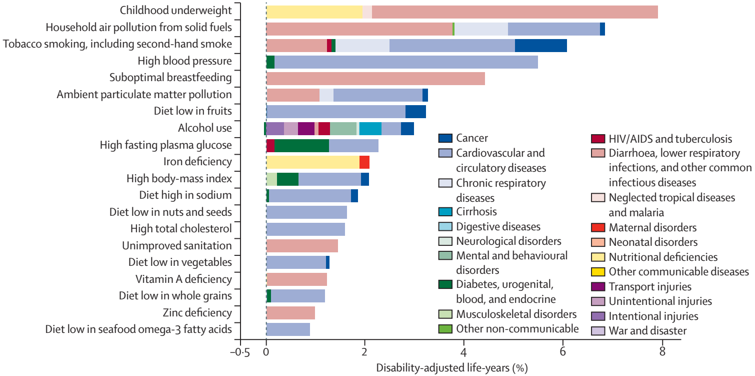
Vos, Flaxman, ..., and Murray; *The Lancet*, 2012

Disability-Adjusted Life Years 2010



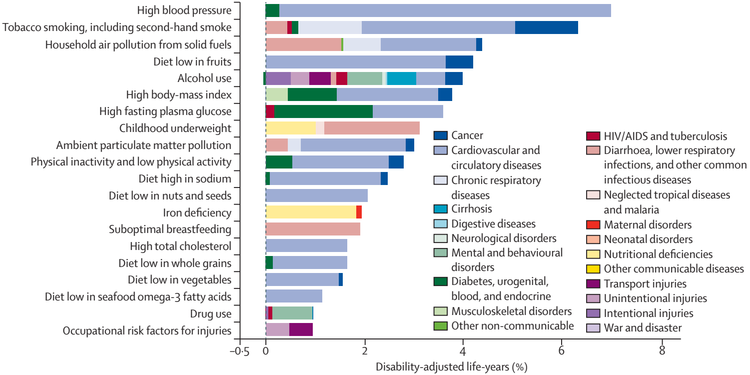
Vos, Flaxman, ..., and Murray; *The Lancet*, 2012

Risk Factors in 1990



Lim, ..., **Finucane** ..., and Ezzati; *The Lancet*, 2012

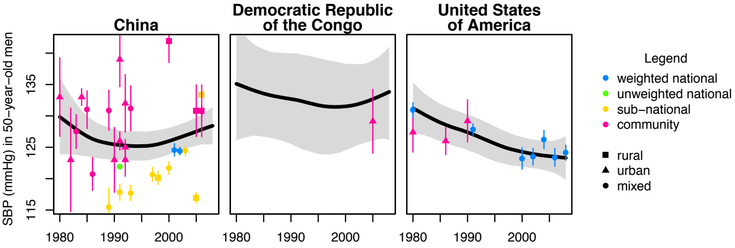
Risk Factors in 2010



Lim, ..., **Finucane** ..., and Ezzati; *The Lancet*, 2012

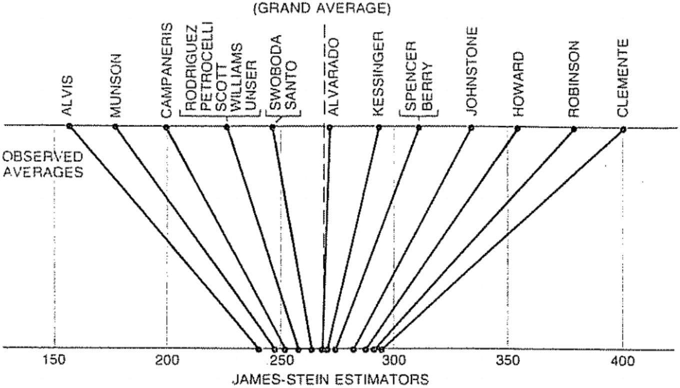
Goal

Synthesize fragmentary data to make country-level estimates of time trends in risk factors by age and sex for all nations.



Outline

Stein's paradox: 'Shrink' raw data toward the mean



JAMES-STEIN ESTIMATORS for the 18 baseball players were calculated by "shrinking" the individual batting averages toward the overall "average of the averages." In this case the grand average is .265 and each of the averages is shrunk about 80 percent of the distance to this value. Thus the theorem on which Stein's method is based asserts that the true batting abilities are more tightly clustered than the preliminary batting averages would seem to suggest they are.

Efron and Morris; *Scientific American*, 1977

Bayesian Estimation of Health Metric Trends

Mariel Finucane

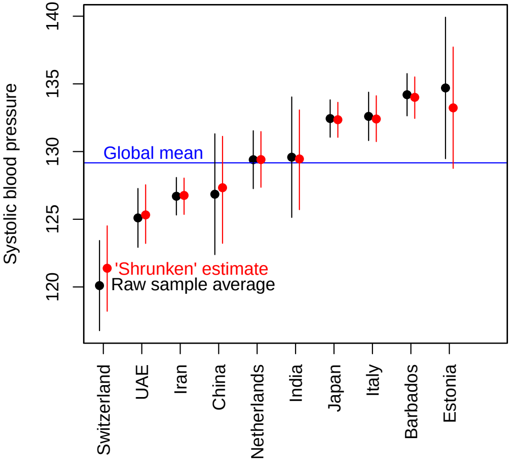
Motivation

'Shrinkage'

Blood Pressure

Childhood Undernutrition

'Shrink' raw data toward the mean with a Bayesian model



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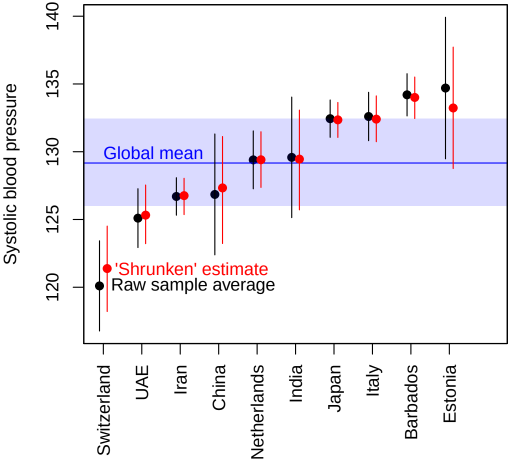
'Shrink' raw data toward the mean with a Bayesian model



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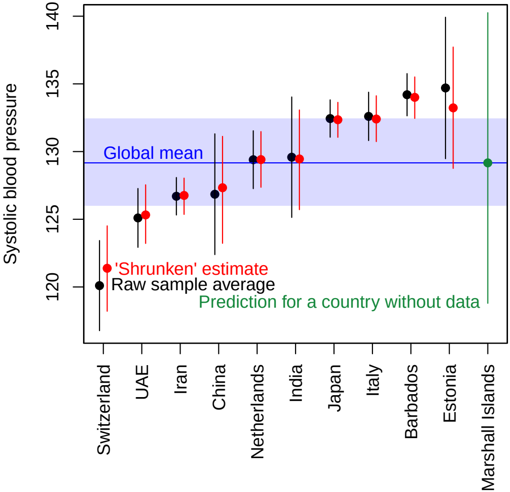
Aa Bb Cc Dd Ee Ff Gg Hh Ii Jj Kk Ll Mm Nn Oo Pp Qq Rr Ss
RULES
Respect your school

Predict for countries without data with a Bayesian model



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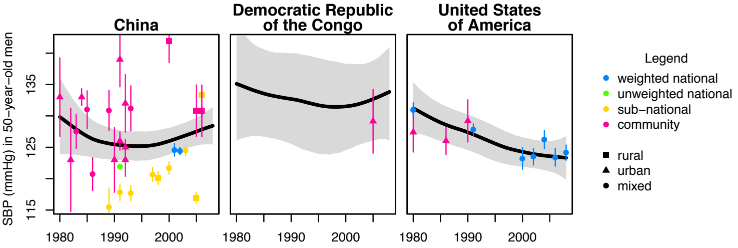
Outline

1. Motivation: Estimating the Global Burden of Disease
2. Bayesian modelling $\Rightarrow$ 'shrinkage'
3. Blood Pressure
4. Childhood Undernutrition

Goal

Synthesize fragmentary data to make country-level estimates of time trends in risk factors by age and sex for all nations.



Finucane, Paciorek, Danaei, and Ezzati; *Statistical Science*, 2014

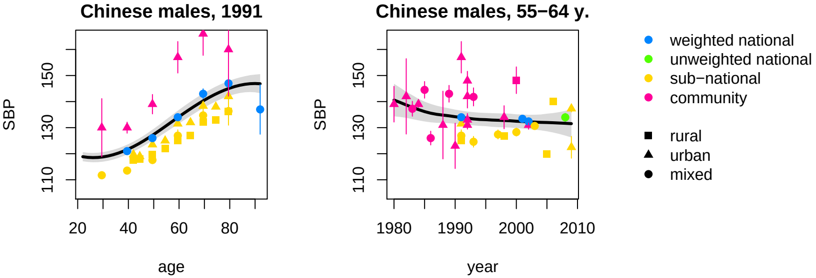
The likelihood

$$y_{h,i} \sim \mathcal{N}\left(\underbrace{a^c_{j[i]} + b^c_{j[i]} t_i}_{\text{geographic hierarchy}} + \overbrace{u_{j[i],t_i}}^{\text{nonlinear change in time}} + \underbrace{\gamma_i(z_h)}_{\text{flexible age model}} + \overbrace{X_i \beta}^{\text{covariate effects}} + \underbrace{e_i}_{\text{random effects}}, \frac{\overbrace{\mathrm{SD}^2_{h,i}}^{\text{sampling var.}}}{n_{h,i}} + \underbrace{\tau^2_i}_{\text{residual var.}} \right)$$

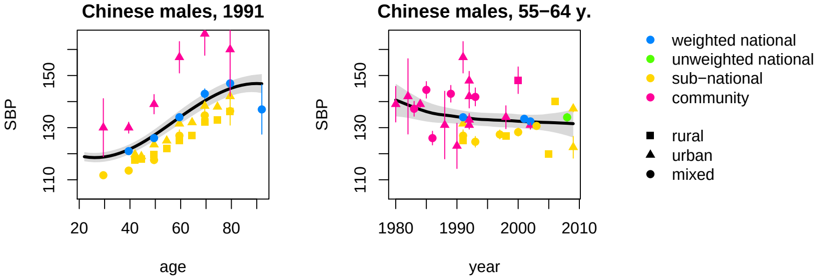
Heteroscedastic random effects ⇒ discount unrepresentative studies

$$y_{h,i} \sim \mathcal{N}\left(\underbrace{a^c_{j[i]} + b^c_{j[i]} t_i}_{\text{geographic hierarchy}} + \overbrace{u_{j[i],t_i}}^{\text{nonlinear change in time}} + \underbrace{\gamma_i(z_h)}_{\text{flexible age model}} + \overbrace{X_i \beta}^{\text{covariate effects}} + \underbrace{e_i}_{\text{random effects}},\ \frac{\overbrace{\mathrm{SD}^2_{h,i}}^{\text{sampling var.}}}{n_{h,i}} + \underbrace{\tau^2_i}_{\text{residual var.}} \right)$$

Heteroscedastic random effects ⇒ discount unrepresentative studies



Heterosedastic random effects $\Rightarrow$ discount unrepresentative studies



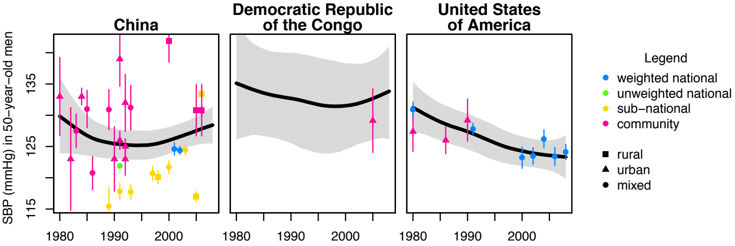
$$\mathrm{Var}(e_i) = \begin{cases} \nu_w & \text{if study } i \text{ is weighted national} \\ \nu_u & \text{if study } i \text{ is unweighted national} \\ \nu_s & \text{if study } i \text{ is sub-national} \\ \nu_c & \text{if study } i \text{ is community,} \end{cases}$$

with $\nu_w < \nu_u < \nu_s < \nu_c$.

Geographic hierarchy ⇒ shrinkage

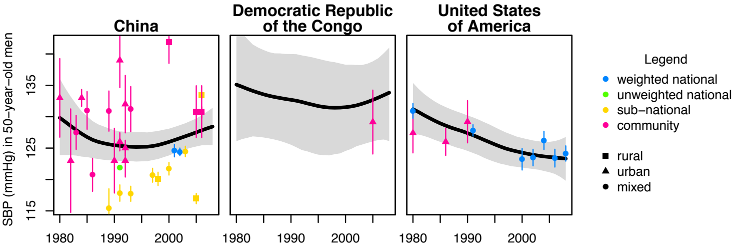
$$y_{h,i} \sim \mathcal{N}\left(\underbrace{a^c_{j[i]} + b^c_{j[i]} t_i}_{\text{geographic hierarchy}} + \overbrace{u_{j[i],t_i}}^{\text{nonlinear change in time}} + \underbrace{\gamma_i(z_h)}_{\text{flexible age model}} + \overbrace{X_i \beta}^{\text{covariate effects}} + \underbrace{e_i}_{\text{random effects}}, \frac{\overbrace{\mathrm{SD}^2_{h,i}}^{\text{sampling var.}}}{n_{h,i}} + \underbrace{\tau^2_i}_{\text{residual var.}} \right)$$

Geographic hierarchy ⇒ shrinkage



Finucane, Paciorek, Danaei, and Ezzati; *Statistical Science*, 2014

Geographic hierarchy $\Rightarrow$ shrinkage



Country: $a_j^c \sim \mathcal{N}(a_{k[j]}^s, \kappa_a^c), \quad b_j^c \sim \mathcal{N}(b_{k[j]}^s, \kappa_b^c).$

Sub-region: $a_k^s \sim \mathcal{N}(a_{l[k]}^r, \kappa_a^s), \quad b_k^s \sim \mathcal{N}(b_{l[k]}^r, \kappa_b^s).$

Region: $a_l^r \sim \mathcal{N}(a^g, \kappa_a^r), \quad b_l^r \sim \mathcal{N}(b^g, \kappa_b^r).$

Finucane, Paciorek, Danaei, and Ezzati; *Statistical Science*, 2014

Gaussian autoregressive priors ⇒ nonlinear change in time

$$y_{h,i} \sim \mathcal{N}\left(\underbrace{a^c_{j[i]} + b^c_{j[i]} t_i}_{\text{geographic hierarchy}} + \overbrace{u_{j[i],t_i}}^{\text{nonlinear change in time}} + \underbrace{\gamma_i(z_h)}_{\text{flexible age model}} + \overbrace{X_i \beta}^{\text{covariate effects}} + \underbrace{e_i}_{\text{random effects}}, \frac{\overbrace{\mathrm{SD}^2_{h,i}}^{\text{sampling var.}}}{n_{h,i}} + \underbrace{\tau^2_i}_{\text{residual var.}} \right)$$

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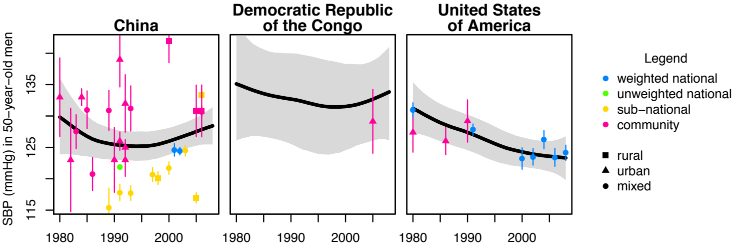
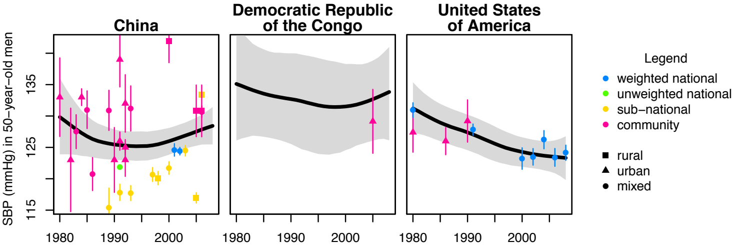
Gaussian autoregressive priors ⇒ nonlinear change in time



Gaussian autoregressive priors ⇒ nonlinear change in time



$$u_j = u_j^c + u_{k[j]}^s + u_{l[k]}^r + u^g,$$

$$u_j^c \sim \mathcal{N}\left(0, (\lambda_c P)^{-1}\right) \quad \text{for } j = 1, \cdots, J$$

$$u_k^s \sim \mathcal{N}\left(0, (\lambda_s P)^{-1}\right) \quad \text{for } k = 1, \cdots, K$$

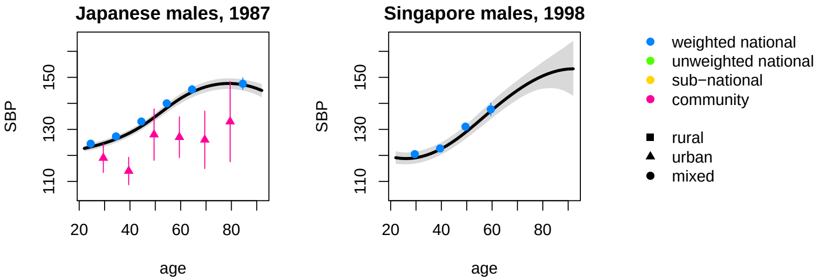
$$u_l^r \sim \mathcal{N}\left(0, (\lambda_r P)^{-1}\right) \quad \text{for } l = 1, \cdots, L$$

$$u^g \sim \mathcal{N}\left(0, (\lambda_g P)^{-1}\right).$$

Cubic splines ⇒ flexible age model

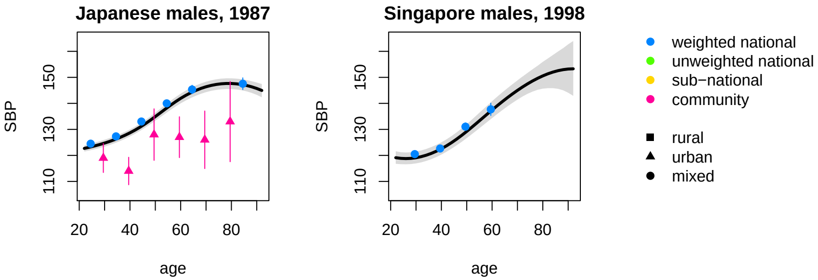
$$y_{h,i} \sim \mathcal{N}\left(\underbrace{a^c_{j[i]} + b^c_{j[i]} t_i}_{\text{geographic hierarchy}} + \overbrace{u_{j[i],t_i}}^{\text{nonlinear change in time}} + \underbrace{\gamma_i(z_h)}_{\text{flexible age model}} + \overbrace{X_i \beta}^{\text{covariate effects}} + \underbrace{e_i}_{\text{random effects}}, \frac{\overbrace{\mathrm{SD}^2_{h,i}}^{\text{sampling var.}}}{n_{h,i}} + \underbrace{\tau^2_i}_{\text{residual var.}} \right)$$

Cubic splines ⇒ flexible age model



Finucane, Paciorek, Danaei, and Ezzati; *Statistical Science*, 2014

Cubic splines $\Rightarrow$ flexible age model



$$\gamma_i(z_h) = \gamma_{1i} z_h + \gamma_{2i} z_h^2 + \gamma_{3i} z_h^3 + \gamma_{4i}(z_h - 45)_+^3 + \gamma_{5i}(z_h - 60)_+^3$$

$$\gamma_{ki} = \psi_k + \phi_k \mu_i + c_{kj[i]}, \;\; k = 1, \ldots, 5$$

$$\mu_i = a_{j[i]}^c + b_{j[i]}^c t_i + X_i \beta + u_{j[i],t_i}$$

Covariates ⇒ better estimates when data are unavailable

$$y_{h,i} \sim \mathcal{N}\left(\underbrace{a^c_{j[i]} + b^c_{j[i]} t_i}_{\text{geographic hierarchy}} + \overbrace{u_{j[i],t_i}}^{\text{nonlinear change in time}} + \underbrace{\gamma_i(z_h)}_{\text{flexible age model}} + \overbrace{X_i \beta}^{\text{covariate effects}} + \underbrace{e_i}_{\text{random effects}}, \frac{\overbrace{\mathrm{SD}^2_{h,i}}^{\text{sampling var.}}}{n_{h,i}} + \underbrace{\tau^2_i}_{\text{residual var.}} \right)$$

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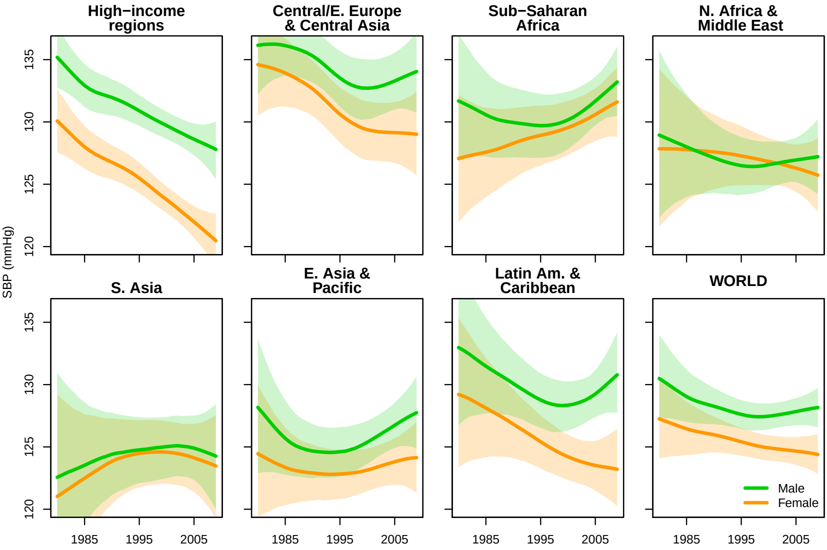
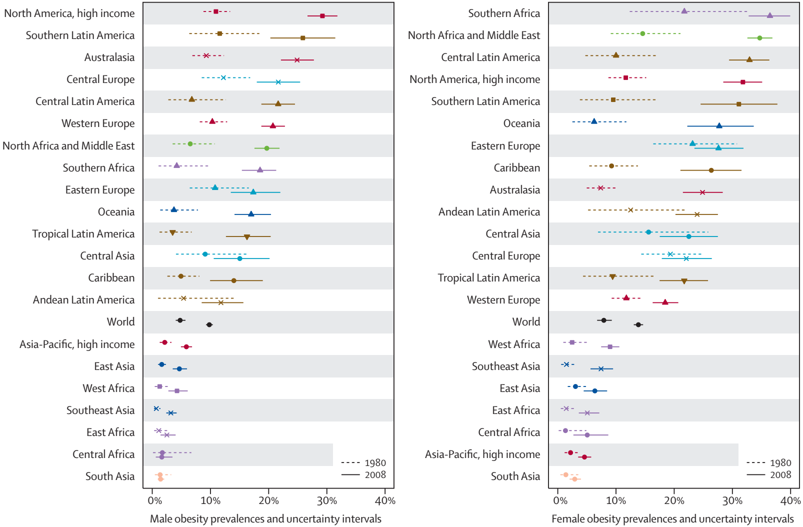
Inference



Danaei*, **Finucane***, ..., and Ezzati; *The Lancet*, 2011

Other model applications: BMI



Finucane*, Stevens*, ..., and Ezzati; *The Lancet*, 2011

Outline

1. Motivation: Estimating the Global Burden of Disease
2. Bayesian modelling ⇒ 'shrinkage'
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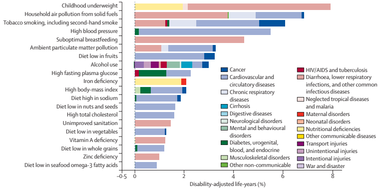
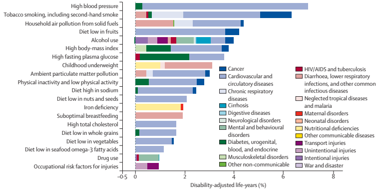
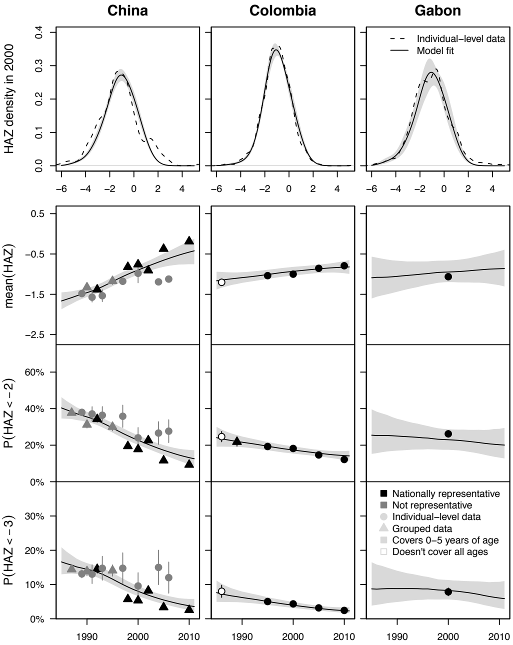
The Global Burden of Disease: Risk Factors in 1990 and 2010





Lim, ..., **Finucane** ..., and Ezzati; *The Lancet*, 2012

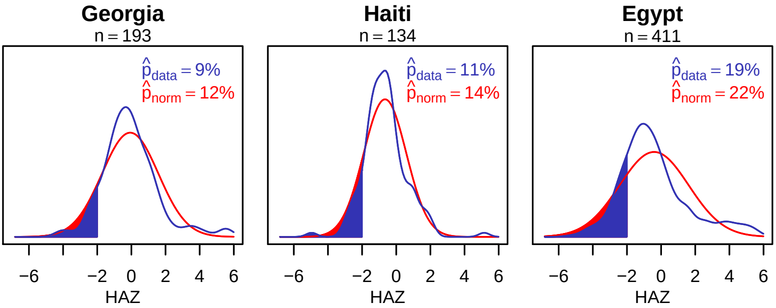


Goal

Synthesize fragmentary data to make country-level estimates of time trends in **the full distribution** of risk factors for all low- and middle-income nations.

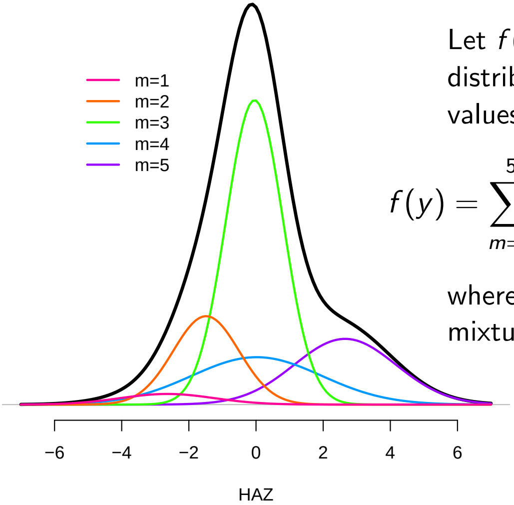
Finucane, Paciorek, Stevens, and Ezzati; *JASA*, in press

Height-for-age z-score distributions are skewed, and assuming normality ⇒ bias



Finucane, Paciorek, Stevens, and Ezzati; *JASA*, in press

Use a mixture of normals ⇒ estimate the shape of distributions



Let $f(y)$ be a distribution of HAZ values:

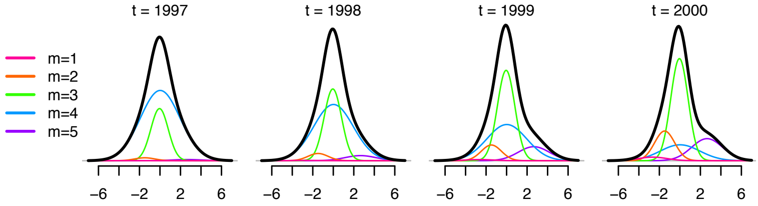
$$f(y) = \sum_{m=1}^{5} w_m \, \mathcal{N}(\mu_m, \sigma_m^2),$$

where m denotes a mixture component.

Allow the weights (w) to vary $\Rightarrow$ a distribution for each study

Let $f_i(y)$ be the distribution of HAZ values in study i:

$$f_i(y) = \sum_{m=1}^{5} w_{mi}\ \mathcal{N}(\mu_m, \sigma_m^2)$$

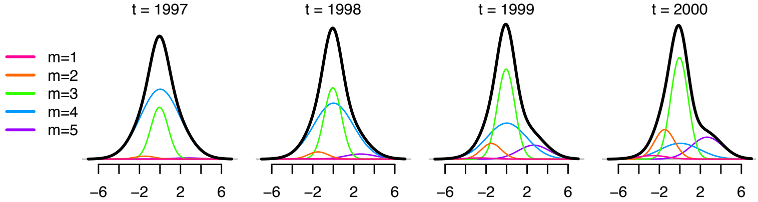


Borrow strength geographically, in time, and in covariates as in the SBP model

$$f_i(y) = \sum_{m=1}^{5} w_{mi}\, \mathcal{N}(\mu_m, \sigma_m^2)$$

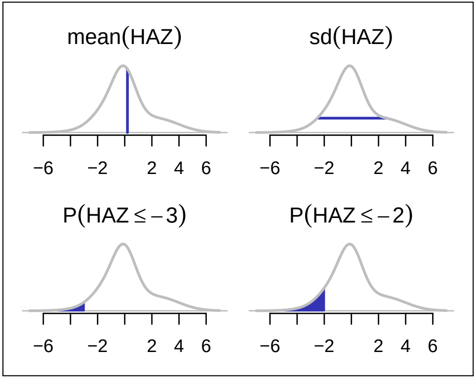
$$w_{mi} = \Phi(\alpha_{mi}) \prod_{u=1}^{m-1} (1 - \Phi(\alpha_{ui}))$$

$$\alpha_{mi} \sim \mathcal{N}(a^c_{mj[i]} + b^c_{mj[i]} t_i + u_{mj[i],t_i} + X_i \beta_m + e_{mi}, \tau^2_{mi})$$



Finucane, Paciorek, Stevens, and Ezzati; *JASA*, in press

Combine individual-level data with summary statistics



Bayesian Estimation of Health Metric Trends

Mariel Finucane

Motivation

'Shrinkage'

Blood Pressure

Childhood Undernutrition

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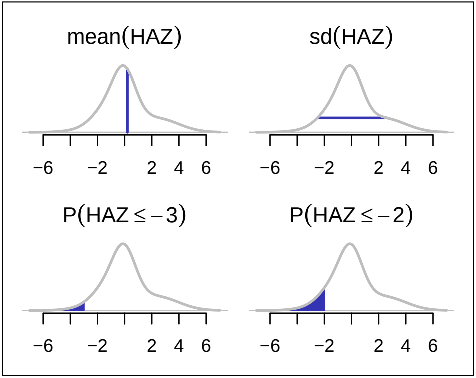
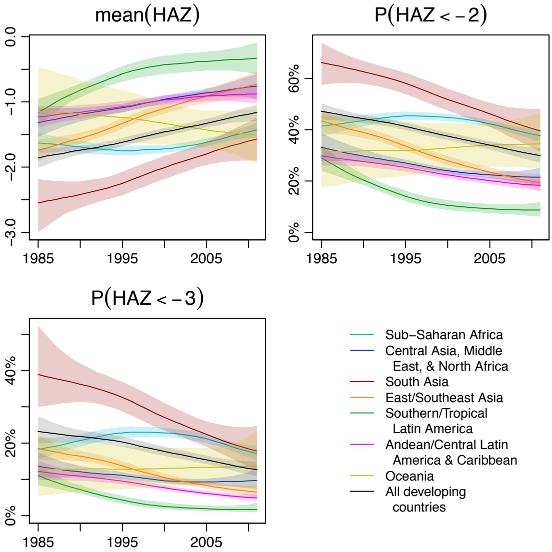
Combine individual-level data with summary statistics



$$P(\theta|y) \propto P(y|\theta)\ P(\theta)$$
$$= P(y_{\text{individual}}|\theta)\ P(y_{\text{summary}}|\theta)\ P(\theta)$$

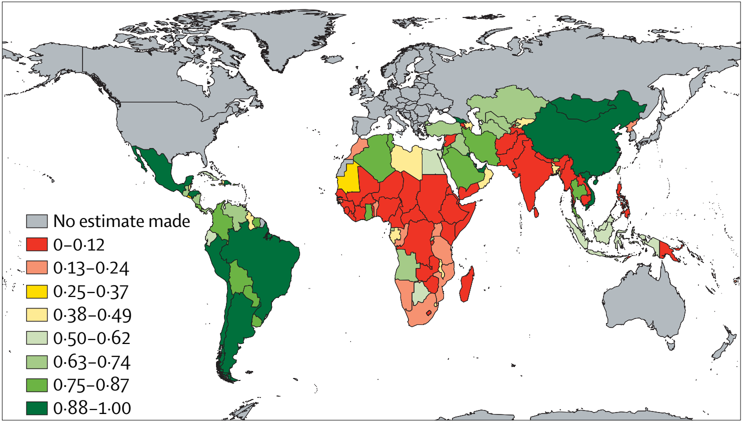
Finucane, Paciorek, Stevens, and Ezzati; *JASA*, in press

Inference



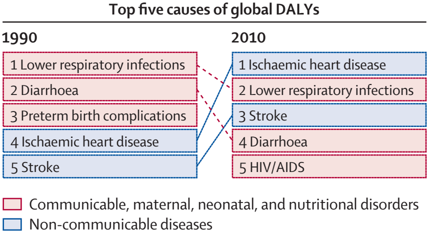
Stevens*, **Finucane***, ..., and Ezzati; *The Lancet*, 2012

Other model applications: Posterior probability of meeting Millennium Development Goal #1



Stevens*, **Finucane***, ..., and Ezzati; *The Lancet*, 2012

A global risk transition



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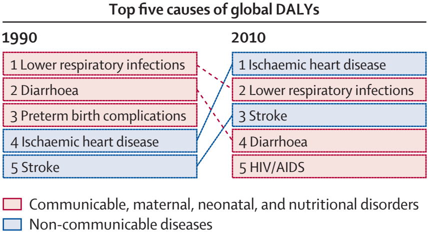
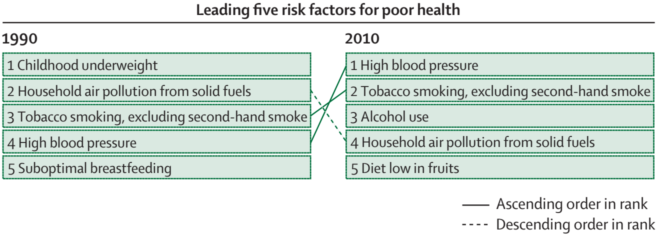
A global risk transition





Why use a Bayesian approach for complex hierarchical analyses?

- To account for hyperparameter uncertainty
- To impose a penalty on model complexity
- To obtain, via MCMC:
    - Computational feasibility
    - Posterior draws for stakeholders
    - Inference on complex functionals

Thanks!

- Chris Paciorek (U.C. Berkeley)
- Majid Ezzati (Imperial College London)
- Gretchen Stevens (WHO)
- Goodarz Danaei (Harvard)

Questions?