Discussion of Talk by Jonathan Wright

William R. Bell, U.S. Census Bureau

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Comment on 2 topics covered by Jonathan Wright:

1. Comparing MSEs of X-11 and canonical ARIMA (SEATS) seasonal adjustments

2. Residual seasonality in NIPA data

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- Reasons for doing indirect seasonal adjustment:
  - potential to better capture seasonality arising from different seasonal patterns in different component series
  - consistency: aggregated SA data is the SA aggregate
- Do the advantages of indirect SA (of GDP) offset the disadvantage of possible residual seasonality that could potentially be avoided by direct SA?

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3. Instead of an ARMAX form as above, why not use

$$y_t = \alpha + \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + z_t \qquad (1 - \rho B) z_t = \varepsilon_t$$

1. Comparing MSEs of X-11 and canonical ARIMA (SEATS) seasonal adjustments

JW Conclusions from Monte Carlo Simulation

- X-13 automatic filter selection tends to select too short seasonal MAs
- Conclusions consistent with other literature
- Model-based SA does better than X-11
- X-11 can get close in some cases
    - But not if θ_{12} is close to zero

Compare and contrast results and conclusions with those of

- Chu, Tiao, and Bell (2012) – for infinite symmetric filters
- Bell, Chu, and Tiao (2012) – for infinite concurrent filters and finite filters

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$$\Rightarrow MSE(\tilde{S}_t) \equiv E[(S_t - \tilde{S}_t)^2] = E[(S_t - \hat{S}_t)^2] + E[(\hat{S}_t - \tilde{S}_t)^2] = g_1 + g_3$$

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- $g_1 = E[(S_t - \hat{S}_t)^2]$ MSE of optimal predictor
- $g_3 = E[(\hat{S}_t - \tilde{S}_t)^2]$ MS difference of $\tilde{S}_t$ from optimal predictor $\hat{S}_t$.

Notes on components of seasonal adjustment MSE

- For $\tilde{S}_t$ a model-based predictor of S_t, g_3 reflects the effects of
  - parameter estimation error
  - model selection error (which changes the canonical decomposition)
- For $\tilde{S}_t$ from X-11 adjustment, g_3 reflects the effects of
  - model selection error and parameter estimation error (affects only forecast extension – minor)
  - difference between X-11 filter and optimal model-based filter
    - find which X-11 filter choice minimizes this error

Notes on components of seasonal adjustment MSE

- Recall that seasonal adjustment MSE is $E[(S_t - \tilde{S}_t)^2] = g_1 + g_3$.
  - For any given model, g_1 is the same for any predictor $\tilde{S}_t$, while g_3 varies with $\tilde{S}_t$
- JW estimates g_3 by simulation
  - reports results on $\sqrt{g_3}$ and ignores g_1
- We ignore g_3 for model-based adjustment, and for X-11 adjustment our g_3 ignores model selection error and parameter estimation error.
  - report MSEs and % differences in MSE between X-11 and optimal model-based adjustment:

$$\text{MSE \% difference } = 100 \times \left(\frac{g_1 + g_3}{g_1} - 1 \right) = 100 \times \left(\frac{g_3}{g_1} \right)$$

  - scaling g_3 by $100/g_1$ aids interpretation of the results

Other differences between the two approaches to comparisons

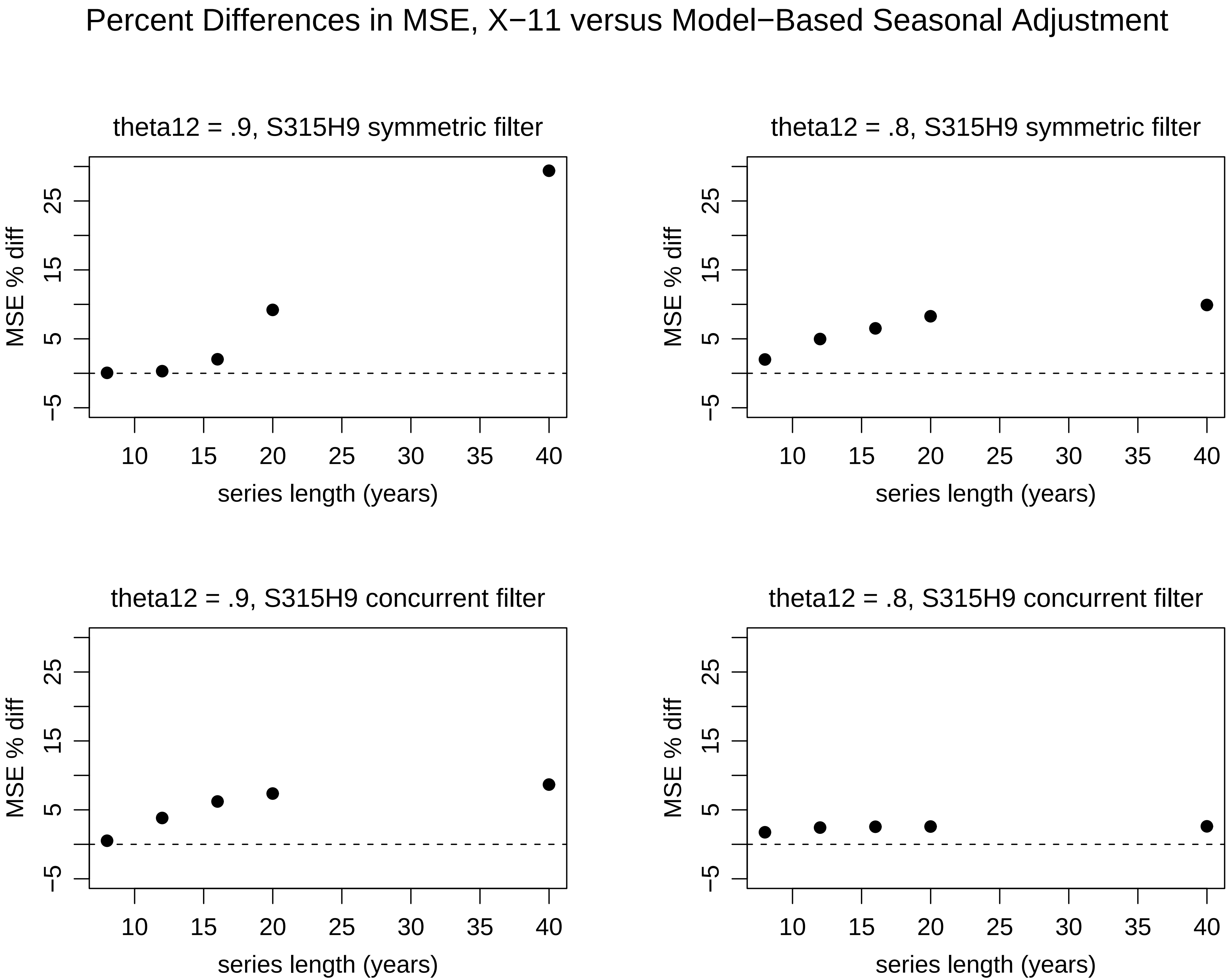
Jonathan Wright	**Bell, Chu, & Tiao**
reports RMSEs	reports MSEs
MSEs calculated by simulation	MSEs calculated by analytical formulas
averages results over $t = 1, \ldots, n$	separate results for $t = n/2$ and $t = n$
??	use full forecast extension for X-11
$n = 120$ (10 years)	results for 8, 12, 16, 20, 40, ∞ years
include X-11 stable seasonal filter	include X-11 3×15 seasonal MA

Comparing MSEs for X-11 and Model-based Filters

Canonical decomposition of the airline model with $\theta_1 = .5$

infinite filter results

	θ_{12}			
	.2	.5	.8	.9
Best X-11 seasonal MA	3×1	3×5	3×15	3×15
MSE % increase for X-11				
symmetric filter	14%	6%	10%	33%
concurrent filter	3%	1%	3%	9%



Percent Differences in MSE, X−11 versus Model−Based Seasonal Adjustment
theta12 = .9, S315H9 symmetric filter
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MSE % diff
series length (years)
−5
5
15
25
10
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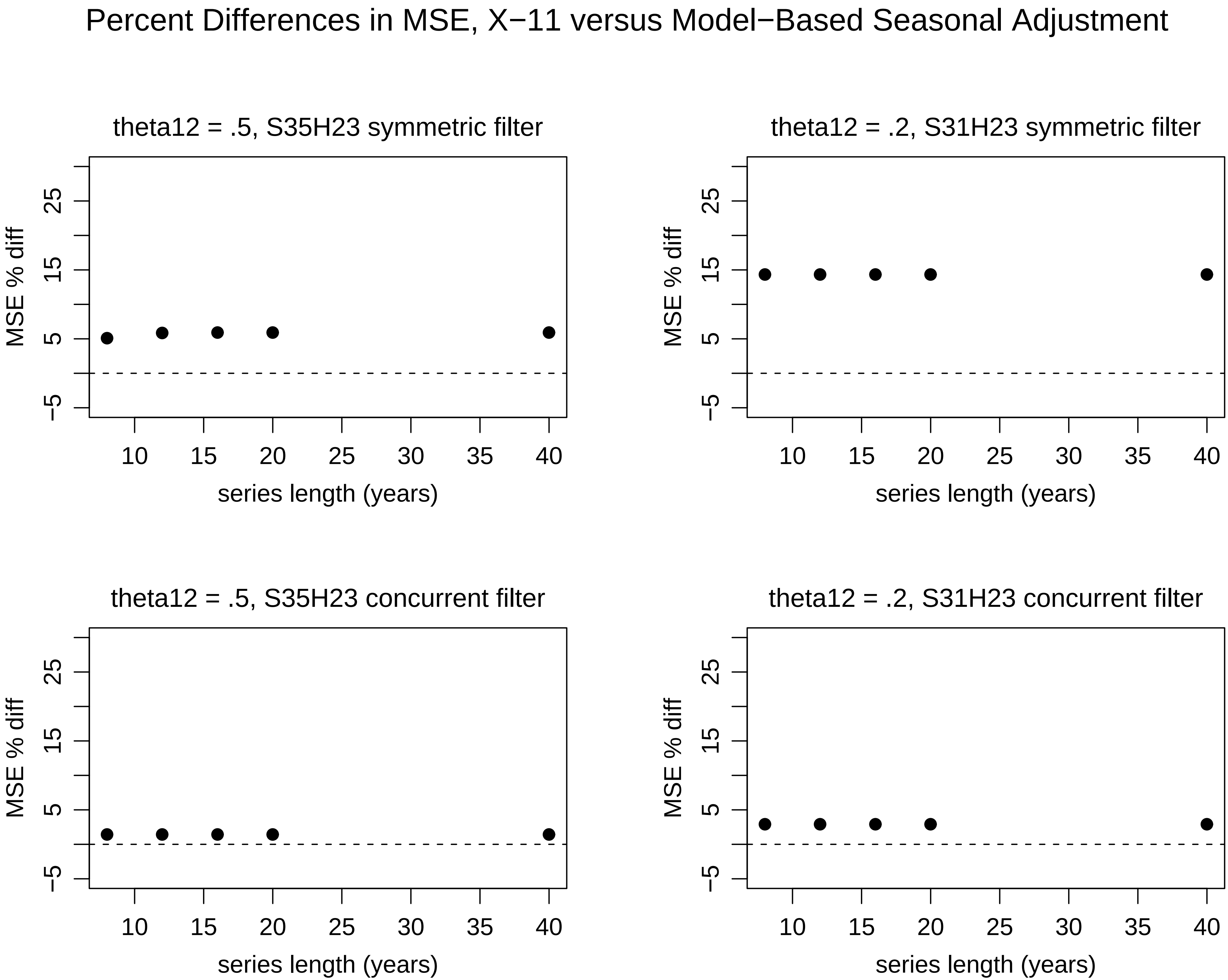
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- The best X-11 filters generally do pretty well for estimating the canonical decomposition, especially for concurrent adjustment or finite sample adjustments with a series that is not long.
- Lone exception where best X-11 filter does poorly: seasonal adjustment in the middle of a very long series when θ_{12} is large (.9).
- Other X-11 filters with a seasonal MA close to the best choice (for example, 3×3 when $\theta_{12} = .5$) have only slightly larger MSEs. X-11 filters far from the best can have larger MSE increases.