Model Selection and Its Important Roles in Surveys

Jiming Jiang

University of California, Davis, USA

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The 2023 Morris Hansen Lecture

Outline

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- This is driven by an unprecedented capability of collecting data, or information.
- Sometimes, there is too much "information";
- yet, in some other cases, there is (still) not enough.
- In any cases, especially in the latter, the idea of "borrowing strength" comes along.

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- Suppose the IQ of students in a particular age group are normally distributed with mean 100 and s.d. 15.
- It is also known that, for a given student, the test scores are normally distributed with mean equal to the student's IQ and s.d. equal to 5.

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- What stories?

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- How to borrow strength?

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- A statistical model allows you to, all of a sudden, known a lot more; but you don't know that for sure.
- Therefore, there is a little bit of "gamble", but it's an educated gamble.

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- "It doesn't matter whether a cat is white or black, as long as it catches mice" — Deng Xiaoping, former Chinese leader
- It doesn't matter whether a statistical model is right or wrong, as long as it helps.

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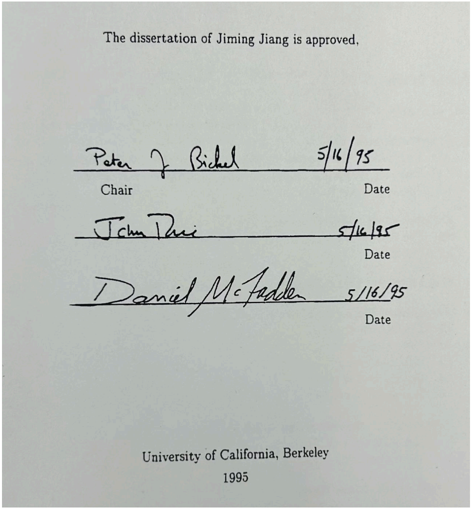
- Example 2. Small area estimation (later).

- Signature page of my Ph. D. thesis (UC Berkeley, 1995)



The dissertation of Jiming Jiang is approved,

Peter J. Bickel 5/16/95

Chair Date

John Rice 5/16/95

Date

Daniel McFadden 5/16/95

Date

University of California, Berkeley

1995

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- Concluded that "careful attention needs to be paid to the development of an appropriate model and its evaluation".
- Rick Valliant (2022) discussed history of explicit models used in sample design and estimation, including an earlier paper of Hansen (1961), in which the author found purely design-based approach was inadequate for analyzing surveys subject to non-responses and other problems.

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- Under an SP model, the finite (real) population is assumed to be a random sample from a "super-population".
- A random sample, Y, from the super-population is assumed to have distribution $p(Y|\theta)$, where the probability distribution, $p(\cdot|\theta)$, is specified under an assumed model (e.g., regression) and θ is the vector of parameters under the model.

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- The posterior inference about the non-sampled part of the population is then carried out via the Bayes' Theorem.

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- Big question: Is model selection a statistical problem?
- Answer: No, it's not.

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- Idea: 1. Build a "statistical fence" to satisfy statistical consideration of model fitting. The fence isolates a subset of candidate models that meet the model-fitting threshold.

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- Such practical considerations can be scientific, economical, legal, or political (e.g., the model must not require privacy-protected information to "train").
- 3. Finally, the threshold of the fence may be determined based on the principle of "letting the data speak".

- Building the fence:

$$Q(M) - Q(M_*) \leq c,$$

where $Q(\cdot)$ is a measure of lack-of-fit, M is a candidate model, and M_* is a (candidate) model that is optimal in terms of model fitting (i.e., one that minimizes Q).

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- Another popular choice of Q is the negative log-likelihood, which applies beyond the linear models. See Jiang & Nguyen (2016) for other examples of Q.

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- Under such a setting, an "over-fitting model", that is, a model that includes the true model as a special case, is also a correct model; it may not be optimal, though, because it can be simplified.
- In particular, M_{f} is a correct model.

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- Thus, once again, $p_c^* = 1$, if c is sufficiently large.
- One is typically looking for the “peak in the middle”.

4. Example

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- This is a classical example of borrowing strength via a statistical model.
- The latter is a linear mixed model (LMM) in the form of

$$y_{ij} = x_{ij}'\beta + v_i + e_{ij},$$

$i = 1, \dots, m$, $j = 1, \dots, n_i$, where ...

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- Why?
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- Nevertheless, the authors also discussed other choices of $x'_{ij}\beta$, such as including the squares and cross product of $x_{ijr}, r = 1, 2$.

- If we consider this as a variable selection problem, the space of candidate predictors may be chosen as

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- The threshold, c, is chosen adaptively as described above.

- The selection results, compared with the BHF models, are presented in the table below:

Outcome Variable	Predictors	
	BHF Model	Adaptive Fence
Corn	Corn & Soybeans	Corn
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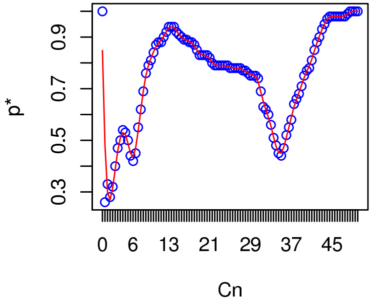
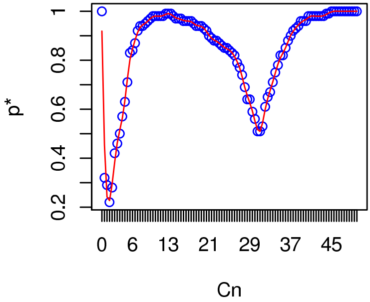
- The plots of p_c^* vs c?

- AF Selection for the Crops Data. Left: p^* vs $c = c_n$ for selecting the corn model. Right: p^* vs $c = c_n$ for selecting the soybeans model.





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- Question: Do you have a practical example of doing the fence this way?
- Answer: I don't.
- What?
- That's why I'm here.
- It would've been nicer to come here with all questions and answers, but it is just as important to have some questions but no answers.

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- Really?
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- We need some genius young people with good computer science training.

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- And, there are much more, if one goes beyond academics.
- Real-life surveys is one of them.

References

- Mood, A. M., Graybill, F. A., and Boes, D. C. (1974), *Introduction to the Theory of Statistics*, 3rd ed., McGraw-Hill, New York.

References

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References

- Mood, A. M., Graybill, F. A., and Boes, D. C. (1974), *Introduction to the Theory of Statistics*, 3rd ed., McGraw-Hill, New York.
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- Fuller, W. A. (2009), *Sampling Statistics*, Wiley

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References

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- Little, R. J. (2008), Weighting and prediction in sample surveys (with discussion), *Calcutta Statist. Assoc. Bull.* 60, 147–194.

References

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- Lahiri, P. (2001), *Model Selection — IMS Lec. Ser.* 38, Institute of Mathematical Statistics.

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- Lahiri, P. (2001), *Model Selection — IMS Lec. Ser.* 38, Institute of Mathematical Statistics.
- Pfeffermann, D. (2013), New important developments in small area estimation, *Statist. Sci.* 28, 40–68.

- Lumley, T, and Scott, A. (2017), Fitting regression models to survey data, *Statist. Sci.* 32, 265–278.

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- Müller, S., Scealy, J. L., and Welsh, A. H. (2013), Model selection in linear mixed models, *Statist. Sci.* 28, 135-167.

- Lumley, T, and Scott, A. (2017), Fitting regression models to survey data, *Statist. Sci.* 32, 265–278.
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- Jiang, J., Rao, J. S., Gu, Z., and Nguyen, T. (2008), Fence method for mixed model selection, *Ann. Statist.* 36, 1669–1692.

- Lumley, T, and Scott, A. (2017), Fitting regression models to survey data, *Statist. Sci.* 32, 265–278.
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- Rao, J. N. K. and Molina, I. (2015), *Small Area Estimation*, 2nd ed., Wiley.

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- Lumley, T, and Scott, A. (2017), Fitting regression models to survey data, *Statist. Sci.* 32, 265–278.
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- Jiang, J. and Nguyen, T. (2016), *The Fence Methods*, World Scientific, Singapore.
- Jiang, J., Nguyen, T. and Rao, J. S. (2009), A simplified adaptive fence procedure, *Statist. Probab. Letters* 79, 625–629.

- Lumley, T, and Scott, A. (2017), Fitting regression models to survey data, *Statist. Sci.* 32, 265–278.
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- Jiang, J., Nguyen, T. and Rao, J. S. (2009), A simplified adaptive fence procedure, *Statist. Probab. Letters* 79, 625–629.
- Battese, G. E., Harter, R. M., and Fuller, W. A. (1988), An error-components model for prediction of county crop areas using survey and satellite data, *J. Amer. Statist. Assoc.* 80, 28–36.