Sample Size Calculations Using R PracTools Package

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Agenda

- Introductions
- Sample size theory
- PracTools package and functions
- Questions / Suggestions

Introductions

- George Zipf
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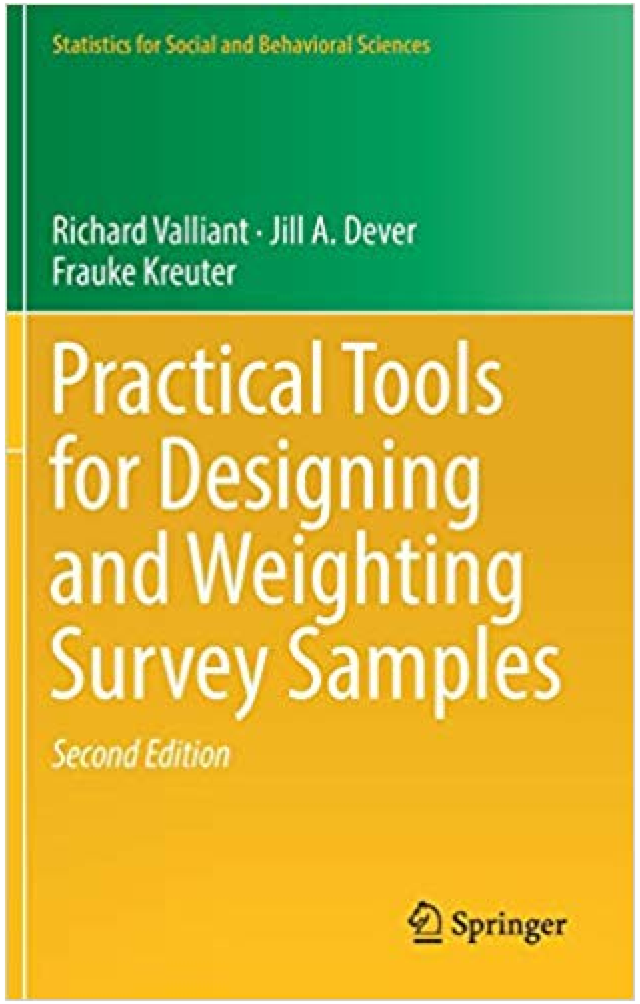
- Dr. Richard Valliant
  - Research Professor Emeritus at Universities of Michigan & Maryland at University of Michigan
  - Author of “Practical Tools for Designing and Weighting Survey Samples”
  - Maintains PracTools package

Practical Tools for Designing and Weighting Survey Samples



Sample Size: Why Sample

- Census is expensive
- Sampling saves time and money and can still have reasonable precision
- Sample size calculations allow the survey designer to define “reasonable precision”

SRS Sample Size: Formula

- Variance formula is: $$V(\bar{y}) = \left(1 - \frac{n}{N}\right)\frac{S^2}{n}$$

- This can be re-written as: $$n = \frac{n_0}{1+\frac{n_0}{N}} \; where \; n_0 = \frac{S^2/_{\bar{y}^2}}{CV_0^2}.$$

- It gets more complicated from here
- This is why we use PracTools

Sample Size: What you need

- N (Population Size)
- α for Confidence Interval
- Variance or Coefficient of Variation (CV)
- Sample unit cost, if these vary
- β, if power is needed

Sample Size constraints

- Standard Error of Estimate
- Coefficient of Variation (CV) = Standard Deviation / Mean
- Margin of Error – Probability within Fixed Bound
- Cost
- Power – Probability of Detectable Difference

PracTools

- If you haven’t already, please bring up R and:

```
install.packages("PracTools")
library("PracTools")
```

PracTools Sample Size Functions

Simple Random Sample (SRS) Design	**PracTools function(s)**
SRS – Continuous variable	nCont, nContMoe
SRS – Proportions	nProp, nPropMoe
SRS – Strata	strAlloc
SRS – Continuous, power required	*power.t.test* (stats package)
SRS – Proportion, power required	*power.prop.test* (stats package)
SRS – 2 stage	clusOpt2

SRS sample size for continuous variable

- Example: You are the primary researcher on a survey about business behavior. While there are many variables of interest, contract amount is a key one. Let's assume that for this population, the average contract is $200,000 and the variance is $980,000,000,000.

nCont(CV0=NULL, V0=NULL, S2=NULL, ybarU=NULL, CVpop=NULL, N=Inf)

```
> nCont(CV0 = 0.30, S2 = 980000000000, ybarU = 200000)
[1] 272.2222
> nCont(CV0 = 0.30, CVpop = sqrt(980000000000)/200000)
[1] 272.2222
```

SRS sample size: N matters

- The finite population correction (fpc) factor can make a big difference. Note that the default is N=Infinity.

```
nCont(CV0=NULL, V0=NULL, S2=NULL, ybarU=NULL, CVpop=NULL, N=Inf)
```

```
> nCont(CV0 = 0.30, S2 = 980000000000, ybarU = 200000)
[1] 272.2222
> nCont(CV0 = 0.30, S2 = 980000000000, ybarU = 200000, N=5000)
[1] 258.1665
```

SRS sample size for continuous variable: MOE

- Let’s say you need to be able to bound the margin of error (MOE). This is good for statements such as “the population value is within 5% of the sample estimate.”
- Expanding on the previous example where average contract is $200,000 and the variance is $980,000,000,000, let’s say you need to be able to bound the margin of error.

```
nContMoe(moe.sw, e, alpha=0.05, CVpop=NULL, S2=NULL, ybarU=NULL, N=Inf)
```

- In the nContMoe function, there are two new parameters: moe.sw, and e

>

SRS sample size for cont. var. – Margin of Error, cont'd

- moe.sw
  - 1 = CI half-width on the variance of the mean; requires S2.
  - 2 = CI half-width on the coefficient of variation; requires CVpop *or* S2 and ybarU
- e is the desired margin of error in percentage terms
  - $0 < e < 1$
- nContMoe examples:

```
> nContMoe(moe.sw=1, e=20000, alpha=0.05, S2=980000000000)
[1] 9411.574
> nContMoe(moe.sw=1, e=50000, alpha=0.05, S2=980000000000)
[1] 1505.852

> nContMoe(moe.sw=2, e=0.10, alpha=0.05, S2=980000000000, ybarU=200000)  # Moe is 10% of 200,000 = 20,000
[1] 9411.574
> nContMoe(moe.sw=2, e=0.25, alpha=0.05, S2=980000000000, ybarU=200000)
[1] 1505.852
```

SRS sample size for proportion

- Functions: nProp and nPropMoe
  - nProp(CV0=NULL, V0=NULL, pU=NULL, N=Inf)
  - nPropMoe(moe.sw, e, alpha, pU, N=Inf)
- nProp examples:

```
> nProp(CV0 = 0.1, pU = 0.5)
[1] 100
> nProp(CV0 = 0.1, pU = 0.1)
[1] 900
> nProp(CV0 = 0.1, pU = 0.9)
[1] 11.11111

> nProp(V0 = 0.0025, pU = 0.5)
[1] 100
> nProp(V0 = 0.0025, pU = 0.1)
[1] 36
> nProp(V0 = 0.0025, pU = 0.9)
[1] 36
```

SRS sample size for proportion, cont'd

- nPropMoe is frequently what people think "should" be the sample size calculation
- Formula is in Cochran, *Sampling Techniques*, 3rd Edition, Section 4.4, pg. 76
- nPropMoe examples:

```
> nPropMoe(moe.sw=1, e=0.10, alpha=0.10, pU = 0.5)
[1] 67.63859
> nPropMoe(moe.sw=1, e=0.10, alpha=0.05, pU = 0.5)
[1] 96.03647
> nPropMoe(moe.sw=1, e=0.05, alpha=0.05, pU = 0.5)
[1] 384.1459

> nPropMoe(moe.sw=2, e=0.10, alpha=0.10, pU = 0.5)
[1] 270.5543
> nPropMoe(moe.sw=2, e=0.10, alpha=0.05, pU = 0.5)
[1] 384.1459
> nPropMoe(moe.sw=2, e=0.05, alpha=0.05, pU = 0.5)
[1] 1536.584
```

SRS sample size for strata

strAlloc(n.tot = NULL, Nh = NULL, Sh = NULL, cost = NULL, ch = NULL, V0 = NULL, CV0 = NULL, ybarU = NULL, alloc=)

- Constraint: n
  - Proportional allocation
  - Equal allocation
  - Neyman allocation
- Cost constrained to total budget
- Precision constrained to overall variance or CV
- strAlloc computes all the above except equal allocation, but requires different inputs
- alloc must be one of “prop”, “neyman”, “totcost”, “totvar”

SRS sample size for strata, cont’d

```
Nh <- c(215, 65, 252, 50, 149, 144)                                    # Strata size
Sh <- c(26787207, 10645109, 6909676, 11085034, 9817762, 44553355)      # Strata SD
ch <- c(1400, 200, 300, 600, 450, 1000)                                # Strata unit cost

# Neyman allocation
strAlloc(n.tot = 100, Nh = Nh, Sh = Sh, alloc = "neyman")

# cost constrained allocation
strAlloc(Nh = Nh, Sh = Sh, cost = 100000, ch = ch, alloc = "totcost")

# allocation with CV target of 0.05
strAlloc(Nh = Nh, Sh = Sh, CV0 = 0.05, ch = ch, ybarU = 11664181, alloc = "totvar")
```

SRS sample size for strata, cont'd

- Here are the results for the Neyman allocation:

```
strAlloc(n.tot = 100, Nh = Nh, Sh = Sh, alloc = "neyman")
 allocation = neyman
               Nh = 215, 65, 252, 50, 149, 144
               Sh = 26787207, 10645109, 6909676, 11085034, 9817762, 44553355
               nh = 34.641683, 4.161947, 10.473487, 3.333804, 8.798970, 38.590108
             nh/n = 0.34641683, 0.04161947, 0.10473487, 0.03333804, 0.08798970, 0.38590108
anticipated SE of estimated mean = 1727173
```

- The output for the other allocation types is similar.

SRS sample size using power calculations

- Functions:
  - power.t.test
  - power.prop.test
- What is required:
  - Delta (Detectable Difference Level) between two populations
  - Probability of obtaining a significant result when the true difference is Delta (Note: Power = $1 - \beta$)
  - Significance Level α
- Part of R stats package

>

power.t.test

- Exactly one of the parameters n, delta, power, sd, and sig.level must be passed as NULL, and that parameter is determined from the others.

```
power.t.test(sd=1, sig.level = 0.05, power=.8, delta=0.1, alternative =
"two.sided", type = "two.sample)
```

```
Two-sample t test power calculation

n = 1570.737

delta = 0.1

sd = 1

sig.level = 0.05

power = 0.8

alternative = two.sided

NOTE: n is number in *each* group
```

```
>
```

power.prop.test

- Exactly one of the parameters n, p1, p2 , power, and sig.level must be passed as NULL, and that parameter is determined from the others.
- Sample size can be function of relative risk, where P1 – P2 = P2(Rel. Risk – 1)

```
power.prop.test(p1 = 0.15, p2 = 0.18, sig.level = 0.05, power=.8,
alternative = "one.sided”)
```

```
Two-sample comparison of proportions power calculation

n = 1891.846

p1 = 0.15

p2 = 0.18

sig.level = 0.05

power = 0.8

alternative = one.sided

NOTE: n is number in *each* group
```

Two-stage sampling

clusOpt2(C1, C2, delta, unit.rv, k=1, CV0=NULL, tot.cost=NULL, cal.sw)

- Parameters
  - Cost
    - C1 = PSU cost
    - C2 = SSU cost
    - tot.cost = total budget
  - CV0 = Target CV
  - delta = homogeneity; 0 < delta < 1
  - unit.rv = unit relvariance
  - k = ratio of *B^2 + W^2* to unit relvariance
  - cal.sw = 1 for fixed total budget and = 2 for target CV0
- delta will affect how sample is optimized over PSUs and SSUs
- Either tot.cost or CV0 must be provided

Two-stage sampling

```
clusOpt2(C1=750, C2=100, delta=0.05, unit.rv=1, k=1, tot.cost=100000, cal.sw=1)
```

C1 = 750

C2 = 100

delta = 0.05

unit relvar = 1

k = 1

cost = 1e+05

m.opt = 51.4

n.opt = 11.9

CV = 0.0502

>

Conclusions:

- We have covered R PracTools functions for sample size calculations.
- For single stage designs, the results of these functions are pretty straightforward.
- If design effect is an issue, remember to multiply the calculated sample size by the design effect for the survey sample size.
- Multi-stage designs are more complicated, but R PracTools does have functions to address the issues.
- It’s always a good idea to discuss survey design with other statisticians.
- And please send us topics for future presentations!

- Questions?