Discussion of Talk by Jonathan Wright

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Disclaimer:

The views expressed here are those of the author and not those of the U.S. Census Bureau.
Comment on 2 topics covered by Jonathan Wright:

1. Comparing MSEs of X-11 and canonical ARIMA (SEATS) seasonal adjustments

2. Residual seasonality in NIPA data
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- Do the advantages of indirect SA (of GDP) offset the disadvantage of possible residual seasonality that could potentially be avoided by direct SA?
Cautions about model used to test for residual seasonality

\[ y_t = \alpha + \rho y_{t-1} + \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + \varepsilon_t \quad t = 1, \ldots, n \]

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3. Instead of an ARMAX form as above, why not use

\[ y_t = \alpha + \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + z_t \quad (1 - \rho B)z_t = \epsilon_t \]
1. Comparing MSEs of X-11 and canonical ARIMA (SEATS) seasonal adjustments

**JW Conclusions from Monte Carlo Simulation**
- X-13 automatic filter selection tends to select too short seasonal MAs
- Conclusions consistent with other literature
- Model-based SA does better than X-11
- X-11 can get close in some cases
  - But not if $\theta_{12}$ is close to zero

Compare and contrast results and conclusions with those of
- Chu, Tiao, and Bell (2012) – for infinite symmetric filters
- Bell, Chu, and Tiao (2012) – for infinite concurrent filters and finite filters
Seasonal adjustment MSE (= estimated seasonal MSE)

\[ y_t = S_t + T_t + I_t \quad t = 1, \ldots, n \]

- Assume a monthly airline model and consider various sets of \((\theta_1, \theta_{12})\) with \(\sigma_a^2 = 1\)
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- \(S_t - \hat{S}_t\) is orthogonal to (uncorrelated with) any function of the data \(\{y_t\}\), including \(\hat{S}_t - \tilde{S}_t\)

\[ \Rightarrow MSE(\tilde{S}_t) \equiv E[(S_t - \tilde{S}_t)^2] = E[(S_t - \hat{S}_t)^2] + E[(\hat{S}_t - \tilde{S}_t)^2] = g_1 + g_3 \]

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where
- \(g_1 = E[(S_t - \hat{S}_t)^2]\) MSE of optimal predictor
- \(g_3 = E[(\hat{S}_t - \tilde{S}_t)^2]\) MS difference of \(\tilde{S}_t\) from optimal predictor \(\hat{S}_t\).
For $\tilde{S}_t$ a model-based predictor of $S_t$, $g_3$ reflects the effects of
- parameter estimation error
- model selection error (which changes the canonical decomposition)

For $\tilde{S}_t$ from X-11 adjustment, $g_3$ reflects the effects of
- model selection error and parameter estimation error (affects only forecast extension – minor)
- difference between X-11 filter and optimal model-based filter
  - find which X-11 filter choice minimizes this error
Recall that seasonal adjustment MSE is $E[(S_t - \tilde{S}_t)^2] = g_1 + g_3$.

For any given model, $g_1$ is the same for any predictor $\tilde{S}_t$, while $g_3$ varies with $\tilde{S}_t$.

JW estimates $g_3$ by simulation:

- reports results on $\sqrt{g_3}$ and ignores $g_1$.

We ignore $g_3$ for model-based adjustment, and for X-11 adjustment our $g_3$ ignores model selection error and parameter estimation error.

- report MSEs and % differences in MSE between X-11 and optimal model-based adjustment:

\[
\text{MSE % difference} = 100 \times \left( \frac{g_1 + g_3}{g_1} - 1 \right) = 100 \times \left( \frac{g_3}{g_1} \right)
\]

- scaling $g_3$ by $100/g_1$ aids interpretation of the results.
Other differences between the two approaches to comparisons

<table>
<thead>
<tr>
<th>Jonathan Wright</th>
<th>Bell, Chu, &amp; Tiao</th>
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<tbody>
<tr>
<td>reports RMSEs</td>
<td>reports MSEs</td>
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<tr>
<td>MSEs calculated by simulation</td>
<td>MSEs calculated by analytical formulas</td>
</tr>
<tr>
<td>averages results over $t = 1, \ldots, n$</td>
<td>separate results for $t = n/2$ and $t = n$</td>
</tr>
<tr>
<td>$n = 120$ (10 years)</td>
<td>use full forecast extension for X-11</td>
</tr>
<tr>
<td>include X-11 stable seasonal filter</td>
<td>results for 8, 12, 16, 20, 40, $\infty$ years</td>
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<td></td>
<td>include X-11 $3 \times 15$ seasonal MA</td>
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Comparing MSEs for X-11 and Model-based Filters

Canonical decomposition of the airline model with $\theta_1 = .5$

<table>
<thead>
<tr>
<th>Infinite filter results</th>
<th>$\theta_{12}$</th>
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<tbody>
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<td>.2</td>
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<tr>
<th>Best X-11 seasonal MA</th>
<th>$3 \times 1$</th>
<th>$3 \times 5$</th>
<th>$3 \times 15$</th>
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<th>MSE % increase for X-11</th>
<th>symmetric filter</th>
<th>concurrent filter</th>
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<tr>
<td>14%</td>
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<td>6%</td>
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Percent Differences in MSE, X−11 versus Model−Based Seasonal Adjustment

theta12 = .9, S315H9 symmetric filter

theta12 = .8, S315H9 symmetric filter

theta12 = .9, S315H9 concurrent filter

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Percent Differences in MSE, X-11 versus Model-Based Seasonal Adjustment

theta1₂ = .5, S35H23 symmetric filter

theta1₂ = .2, S31H23 symmetric filter

theta1₂ = .5, S35H23 concurrent filter

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Summary of Conclusions from CTB and BCT

- Length of best X-11 seasonal MA increases with $\theta_{12}$. X-13 automatic filter selection sometimes picks shorter seasonal MAs than the best (assessed in a small simulation study).
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Other X-11 filters with a seasonal MA close to the best choice (for example, 3 x 3 when $\theta_{12} = .5$) have only slightly larger MSEs. X-11 filters far from the best can have larger MSE increases.